QP CODE 211122

PILLAI COLLEGE OF ENGINEERING, NEW PANVEL
(Autonomous) (Accredited 'A+' by NAAC)
END SEMESTER EXAMINATION
FIRST HALF 2022(Supplementary)

SEM IV **BRANCH: INFORMATION TECHNOLOGY**

Subject:- Engineering Mathematics IV
Time: 02.00 Hours
Max. Marks: 60
Date: 18/07/2022
Subject Code: IT207

N.B 1. Q.1 is compulsory
2. Attempt any two from the remaining three questions
3. Each Question carry 20 marks.

		Marks
Q.1.	**Attempt All**	
a)	The Probability density function of a random variable X is: (see table below) Find $(i) P(X<4)$ $(ii) P(3<X\leq 6)$	5
b)	Using Euclid's algorithm find x and y satisfying, $\gcd(2378,1769)=2378x+1769y$	5
c)	Check whether the vectors are linearly dependent, if so, find their relation: $(0,3,1,-1),(6,0,5,1),(4,-7,1,3)$	5
d)	Show that the set of all divisions of 70 form a lattice.	5
Q.2.	**Attempt All**	
a)	In a Poisson distribution $P(X=3)$ is $2/3$ of $P(X=4)$. Find the mean and the standard deviation.	4
b)	Find the complete solution of $a_n+2a_{n-1} = n+3$ for $n\geq 1$ with $a_0=3$.	4
c)	Calculate the Correlation Coefficient from the following data: (see table below)	6
d)	Solve the following system of congruences. $x\equiv 5(mod\ 6),\quad x\equiv 4(mod\ 11),\ \ x\equiv 3(mod\ 17)$	6

Q.1 a):

X	0	1	2	3	4	5	6
P(X=x)	k	3k	5k	7k	9k	11k	13k

Q.2 c):

X	23	27	28	29	30	31	33	35	36	39
Y	18	22	23	24	25	26	28	29	30	32

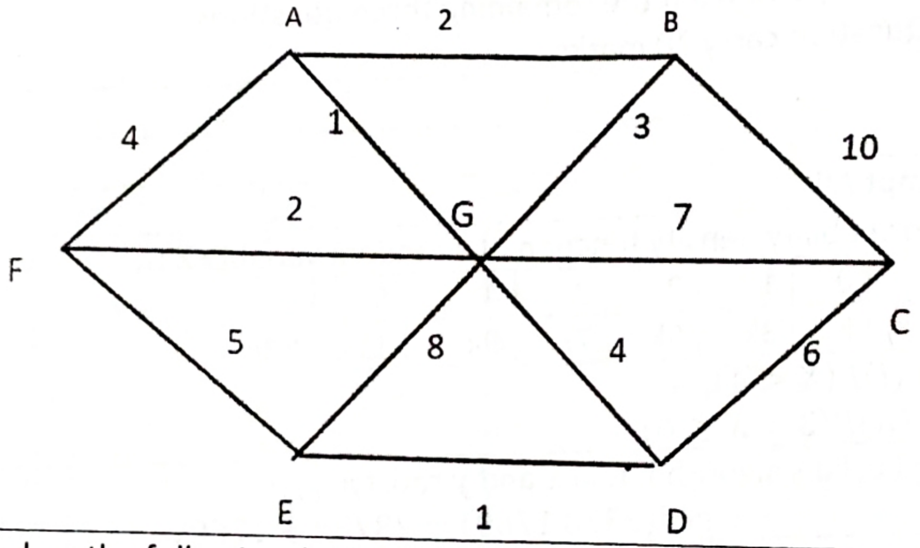
P.T.O.

Q.3. **Attempt All**

a) The incomes of a group of 10,000 persons were found to be normally distributed with mean rupees 520 and standard deviation rupees 60. Find the number of persons having incomes between rupees 400 and 550. **4**

b) State Fermat's theorem and hence find 2^{-1}(mod 31). **4**

c) Using Kruskal's Algorithm construct the minimum spanning tree of the following graph **6**



d) Based on the following data determine if there is a relation between literacy and smoking. **6**

	Smokers	Non-smokers
Literates	83	57
Illiterates	45	68

Q.4. **Attempt All**

a) (i) Is every Hamiltonian graph Eulerian? Give an example.
(ii) Is every Eulerian graph Hamiltonian? Give an example. **4**

b) The equations of two lines of regression are $x = 19.13 - 0.87y$ and $y = 11.64 - 0.50x$. Find the regression coefficients. **4**

c) Construct an orthonormal basis of R^3 using Gram-Schmidt process: **6**

$$S = \{(3,0,4), (-1,0,7), (2,9,11)\}$$

d) A drug was administered to 10 patients and the changes in the sugar content in the blood was recorded as under 10, 8, -6, -4, 2, -8, 6, -5, -3, -6. Is it reasonable to believe that the drug has no effect on change of sugar? **6**

$\boxed{\text{Q. 1 (a)}}$ The p.d.f. is

X	0	1	2	3	4	5	6	Total
$P(X=x)$	k	3k	5k	7k	9k	11k	13k	49k

To find ~~k~~ i) k & (ii) $P(3 < X \leq 6)$

$P(X<4)$

$$\Sigma P(X=x_i) = 1$$

$$49k = 1 \Rightarrow k = \frac{1}{49} \quad \therefore$$

X	0	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

$\therefore$

$$\therefore \text{(i) } P(X<4) = P(X=0)+P(X=1)+P(X=2)+P(X=3)$$

$$= k+3k+5k+7k$$

$$= 16k$$

$$= 16\left(\frac{1}{49}\right)$$

$$\therefore P(X<4) = \frac{16}{49}$$

$$\text{(ii) } P(3<X\leq 6) = P(X=4)+P(X=5) + P(X=6)$$

$$= 9k+11k+13k$$

$$= 33k$$

$$= 33\left(\frac{1}{49}\right)$$

$$= \frac{33}{49}$$

$\boxed{Q \cdot 1(b)}$ Using Euclid's algorithm to find x and y satisfying

$$\gcd(2378, 1769) = 2378x + 1769y$$

By Euclid's algorithm, we have

$$2378 = 1769 + 609 \quad \text{—(i)}$$

$$1769 = (2)(609) + 551 \quad \text{—(ii)}$$

$$609 = (1)(551) + 58 \quad \text{—(iii)}$$

$$551 = (9)(58) + 29 \quad \text{—(iv)}$$

$$\boxed{58 = (2)(29) + 0} \Rightarrow \gcd(2378, 1769) = 29$$

$$1769\overline{)2378}^{\,1}$$
$$1769$$

$$609\overline{)1769}^{\,2}$$
$$1218$$

$$551\overline{)609}^{\,1}$$
$$-551$$

$$58\overline{)551}^{\,9}$$
$$522$$

$$29\overline{)58}^{\,2}$$
$$58$$
$$00$$

$$29\overline{)58}^{\,2}$$
$$58$$
$$00$$

$\therefore \boxed{\gcd(2378, 1769) = 29}$

$\therefore$ By using backsubstitution, we have

$$29 = 551 - (9)(58) \quad \text{By (iv)}$$

$$= 551 - 9(609 - 551) \quad \text{by (iii)}$$

$$= (-9)(609) + (10)(551)$$

$$= (-9)(609) + (10)(1769 - (2)(609)) \quad \text{by (ii)}$$

$$= (-9)(609) + (10)(1769 - (2)(609)) \quad (\text{by (ii)})$$

$$= (-29)(609) + (10)(1769)$$

$$= (-29)(2378 - 1769) + (10)(1769) \quad (\text{by (i)})$$

$$= (-29)(2378) + (39)(1769)$$

Therefore, expressed as a linear combination

$$29 = \gcd(2378, 1769) = 2378x + 1769y$$

$$x = -29, \quad y = 39$$

$\boxed{Q.1(c)}$

Given vectors

$$u = (0,3,1,-1)$$

$$v = (6,0,5,1)$$

$(4,-7,1,3) = (4,-7,1,3) = w$ (say)

– To check whether u, v, w are linearly independent; & if so to find their relation

$\therefore$ Consider $k_1 u + k_2 v + k_3 w = 0$

$$k_1(0,3,1,-1) + k_2(6,0,5,-1) + (k_3)(4,-7,1,3) = (0,0,0,0) = 0$$

$$\therefore (0, 3k_1, k_1, -k_1) + (6k_2, 0, 5k_2, -k_2) + (4k_3, -7k_3, k_3, 3k_3) = (0,0,0,0)$$

$$\therefore (6k_2 + 4k_3, 3k_1 + 0 - 7k_3, k_1 + 5k_2 + k_3, -k_1 - k_2 + 3k_3) = (0,0,0,0)$$

$$\therefore \left.\begin{aligned} 0k_1 + 6k_2 + 4k_3 &= 0 \\ 3k_1 + 0k_2 - 7k_3 &= 0 \\ k_1 + 5k_2 + k_3 &= 0 \\ -k_1 - k_2 + 3k_3 &= 0 \end{aligned}\right\} \Rightarrow \begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = AX = 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ①$$

which is Homogeneous system of Linear Equations

$$\therefore A \simeq \begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ -1 & 1 & 3 \end{bmatrix} \overset{R_4+R_3 \sim R_4}{\sim} \begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ 0 & 6 & 4 \end{bmatrix} \overset{R_4-R_1\to R_4}{\sim} \begin{bmatrix} 0 & 6 & 4 \\ 3 & 0 & -7 \\ 1 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\overset{R_{13}}{\sim} \begin{bmatrix} 1 & 5 & 1 \\ 3 & 0 & -7 \\ 0 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_2-3R_1\to R_2}{\sim} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 6 & 4 \\ 0 & 0 & 0 \end{bmatrix} \overset{R_3+\frac{1}{5}R_2\to R_3}{\sim} \begin{bmatrix} 1 & 5 & 1 \\ 0 & -15 & -10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ by ① we get

Q.1

$$\begin{bmatrix} ① & 5 & 1 \\ 0 & \textcircled{-15} & -10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \overset{R_1+\frac{1}{3}R_2 \to R_1}{\sim} \begin{bmatrix} 1 & 0 & -\frac{7}{3} \\ 0 & -15 & -10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} X = 0$$

which is in Echelon form,

$\therefore k_1+5k_2+k_3=0$ — ① $\Big\} \Rightarrow$ $m=2$ eq^n's in $3=n$ variable

$-15k_2-10k_3=0$ — ② & $n>m$

$\therefore$ Given system of Equations has infinite solution.

Let free variable $= k_3 = t$

by ① $\therefore -15k_2-10k_3=0 \Rightarrow -15k_2-10t=0$

$\Rightarrow k_2 = -\frac{2t}{3}$

by ②

$\therefore k_1+5k_2+k_3=0$

$\Rightarrow k_1+5\left(-\frac{2t}{3}\right)+t=0$

$\Rightarrow k_1-\frac{7t}{3}=0$

$\therefore k_1=+\frac{7}{3}t \Rightarrow$

$\therefore t=3$ implies

$k_1=\frac{7}{3}t = 7$

$k_2=-\frac{2}{3}t = -2$

$k_3=t = 3$

$\Rightarrow t=3 \Rightarrow k_1=7, k_2=-2$

$k_3=3$

$\therefore k_1u+k_2v+k_3w=0$

$\therefore 7u-2v+3w=0$

$7u-2v+3w=0$

$\therefore (7)(0,3,1,-1)+(-2)(6,0,5,1)+3(4,-7,1,3)=(0,0,0,0)$

$$\boxed{7u-2v+3w=0}$$

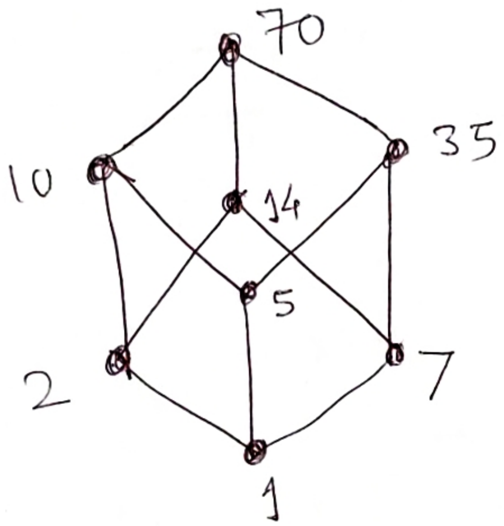
i.e. $(7)(0,3,1,-1)+(-2)(6,0,5,1)+(3)(4,-7,1,3)=(0,0,0,0)$

Q.1(d) Show that the set of all divisors of 70 form a lattice.

D_{70} = Divisors of 70

= { 1, 2, 5,7, 10, 14, 35, 70 }

= { 1, 2, 5, 7, 10, 14, 35, 70 }



D_{70}

for $x, y \in D_{70}$, $x \vee y = \text{l.u.b.}\{x, y\}$ &
$= \text{lcm}\{x, y\}$ exists

$x \wedge y = \text{g.l.b.}\{x, y\}$ exists
$= \text{gcd}\{x, y\}$ exists

~~Here~~ . The Following Table shows $x \vee y$

y \ x	1	2	5	7	10	14	35	70
1	1	2	5	7	10	14	35	70
2	2	2	10	14	10	14	70	70
5	5	10	5	35	10	70	35	70
7	7	14	35	7	70	14	35	70
10	10	10	10	70	10	70	70	70
14	14	14	70	14	70	14	70	70
35	35	70	35	35	70	70	35	70
70	70	70	70	70	70	70	70	70

Also prepare table for $x \wedge y$.

Q.2 (a) Given In a Poisson Distribution

$P(X=3) = \frac{2}{3}$ of $P(X=4)$

To find mean and standard deviation

$$\therefore P(X=3) = \frac{2}{3} \times P(X=4) \quad \text{——}(\alpha)$$

$$P(X=x) = \frac{e^{-m} m^x}{x!}, \quad m = \text{mean}$$

$$P(X=3) = \frac{e^{-m} m^3}{3!}$$

$$P(X=4) = \frac{e^{-m} . m^4}{4!}$$

$\therefore$ By eqn (α)

$$P(X=3) = \frac{2}{3} \times P(X=4)$$

$$\frac{\cancel{e^{-m}} m^3}{3!} = \frac{2}{3} \times \frac{\cancel{e^{-m}} m^4}{4!}$$

$$\frac{\cancel{m^3}}{\cancel{3!}} = \frac{2}{3} \times \frac{\cancel{m^3} . m}{4 \times (\cancel{3!})}$$

$$1 = m \times \frac{2}{3} \times \frac{1}{4} = \frac{m}{6}$$

$$\therefore \boxed{m = 6}$$

For poisson Distribution

mean = variance

$\therefore$ variance = mean = m = 6

& variance = $(\text{standard deviation})^2 = \sigma^2$

$\therefore \sigma^2 = 6 \quad \therefore \sigma = \sqrt{6} = 2.499489$

$\therefore$ mean = m = 6, standard deviation = $\sigma = 2.499489$

Q.2(b) To find the complete solution of

$a_n + 2a_{n-1} = n+3$, for $n \geq 1$ ——— ①

with $a_0 = 3$

Substitute $a_n = x^n$ in Homogeneous part of ①

$a_n + 2a_{n-1} = 0$

$\therefore x^n + 2x^{n-1} = 0$

$\therefore x^{n-1} \cdot x + 2x^{n-1} = 0$

$\therefore x+2 = 0 \quad \therefore x = (-2)$

$\therefore a_n = $ C.F. + P.I. = C.F. + $a_n(p) = a_n^c + a_n^p$

C.F. $= \alpha(-2)^n = \alpha(-2)^n$

RHS of ① is $n+3$

$\therefore$ P.I. $= a_n(p) = An + B = a_n$

$a_{n-1} = A(n-1) + B$

$\therefore$ by ①

$a_n + 2a_{n-1} = n+3$

$An + B + 2A(n-1) + 2B = n+3$

$An + B + 2An - 2A + 2B = n+3$

$n(A+2A) + (3B - 2A) = n+3$

$\therefore n(3A) + (3B-2A) = \cancel{0} n+3 = n+3 = 1 \cdot n + 3$

$\therefore 3A = 1$ & $3B - 2A = 3$

$\therefore A = \frac{1}{3}$ & $3B - \frac{(2)}{1}\left(\frac{1}{3}\right) = 3$

$3B = 3 + \frac{2}{3} = \frac{11}{3}$

$B = \frac{11}{9}$

$\therefore a_n^p = \frac{n}{3} + \frac{11}{9}$

$\therefore$ Complete solution of ① is $a_n = \alpha(-2)^n + \frac{n}{3} + \frac{11}{9}$

$a_0 = 3 \Rightarrow n=0$ implies $a_0 = \alpha \cdot 1 + 0 = 3 \quad \therefore \alpha = 3 \quad \therefore \boxed{a_n = 3(-2)^n + \frac{n}{3} + \frac{11}{9}}$

$\boxed{Q.2(c)}$ To calculate the correlation coefficient (r) from the following Data

X	23	27	28	29	30	31	33	35	36	39
Y	18	22	23	24	25	26	28	29	30	32

X	Y	$dx = X - \bar{X}$ $X - 31.1$	$dy = Y - \bar{Y}$ $Y - 25.7$	$dx\,dy$	dx^2	dy^2
23	18	−8.1	−7.7	0 62.37	65.61	59.29
27	22	−4.1	−3.7	15.17	16.81	13.69
28	23	−3.1	−2.7	8.37	9.61	7.29
29	24	−2.1	−1.7	3.57	4.410	2.89
30	25	−1.1 ~~0.1~~	−0.7	0.77	1.21	0.49
31	26	~~0.9~~ −0.1	0.3	−0.03	0.01	0.09
33	28	1.9	2.3	4.37	3.61	5.29
35	29	3.9	3.3	12.87	15.21	10.89
36	30	4.9	4.3	21.07	18.49	21.07
39	32	7.9	6.3	49.77	39.69	47.77
311	257			Sum = ~~[illegible]~~ 178.3	Sum = 158.1	Sum = 202.9

n = 10

~~$Q = \frac{\Sigma X}{n}$~~

$$\bar{X} = \frac{\Sigma X}{n} = \frac{311}{10} = 31.1$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{257}{10} = 25.7$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{[\Sigma(x-\bar{x})^2][\Sigma(y-\bar{y})^2}} = \frac{178.3}{\sqrt{(202.9)(158.1)}}$$

$$r = 0.9955 \quad \therefore \boxed{r = 0.9955}$$

Q. 2(d) To solve the following system of congruences

$x \equiv 5 (\bmod 6)$
$x \equiv 4 (\bmod 11)$
$x \equiv 3 (\bmod 17)$

Answer. $x \equiv 785 (\bmod 1122)$

1	1	2	2
-	7	8	5

let $m = 6 \cdot 11 \cdot 17 = 1122$
$a_1 = 5, \quad a_2 = 4, \quad a_3 = 3$
$\therefore x = a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$

where $m_1 = \frac{m}{6} = 187$, $m_2 = \frac{m}{11} = 102$,

$m_3 = \frac{m}{17} = 66$

Now modulo 6, $y_1 = 187^{-1} (\bmod 6) = 187^{-1} (\bmod 6)$
$= 1$ as $187 \cdot 1 \equiv 1 (\bmod 6)$

Modulo 11, $y_2 = 102^{-1} (\bmod 11) = 4$ as $102 \cdot 4 \equiv 1 (\bmod 11)$

Modulo 17, $y_3 = 66^{-1} (\bmod 17) = 8$ as $(66 \cdot 8) \equiv 1 (\bmod 17)$

$x \equiv a_1 M_1 y_1 + a_2 M_2 y_2 + a_3 M_3 y_3$
$\equiv 5 \times 187 \times 1 + 4 \times 102 \times 4 + 3 \times 66 \times 8$
$\equiv 4151 (\bmod 1122) = 4151 (\bmod 1122)$

$$\boxed{x = 785 (\bmod 1122)}$$

Q.3(9) The incomes of a group of 10,000 persons were found to be normally distributed with mean rupees 520 & standard deviation rupees 60.

To find the number of persons having incomes between rupees 400 and 550

$\therefore m = 520, \quad \sigma = 60$

$$Z = \frac{X - m}{\sigma} = \frac{X - 520}{60}$$

$\therefore$ When $X = 400$, $Z = \frac{400 - 520}{60} = \frac{-120}{60} = -2$

when $X = 500$, $Z = \frac{5500 - 520}{60} = +\frac{30}{60} = +\frac{1}{2} = +0.5000$

$= 0.5000$

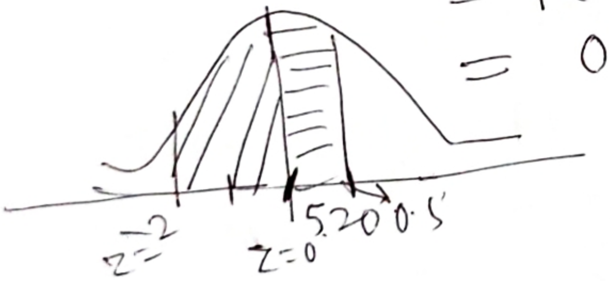
$$\therefore P(400 < X < 500) = P(-2 < Z < 0.5)$$

$$= P(-2 < Z < 0) + P(0 < Z < 0.5)$$

$$= P(0 < Z < 2) + P(0 < Z < 0.5)$$

$$= 0.4772 + 0.1915$$

$$= 0.6687$$



$\therefore$ The Number of persons having incomes between rupees 400 and 550 is

$10000 \times P(400 < X < 500)$

$= 10000 \times 0.6687$

$= 6687$

$\boxed{\text{Q. 3(h)}}$ To state Fermat's theorem & hence to find 2^{-1} (mod 31)

~~Def~~ [illegible]

If a, ~~p~~ are integers with $n \nmid a$ $(\gcd(a,n)=1)$ ~~n~~ and p is prime

Then ~~a ≡~~

If p is a prime & a is integer such that $p \nmid a$ $(\gcd(a,p)=1)$, Then

$a^{p-1} \equiv 1$ (modulo p). Here 31 prime & $31 \nmid 2$

$\Rightarrow$ $2^{30} \equiv 1$ (modulo 31)

∴ To find 2^{-1} (mod 31)

i.e. $y = 2^{-1}$ (mod 31)

Then $2y \equiv 1$ (mod 31)

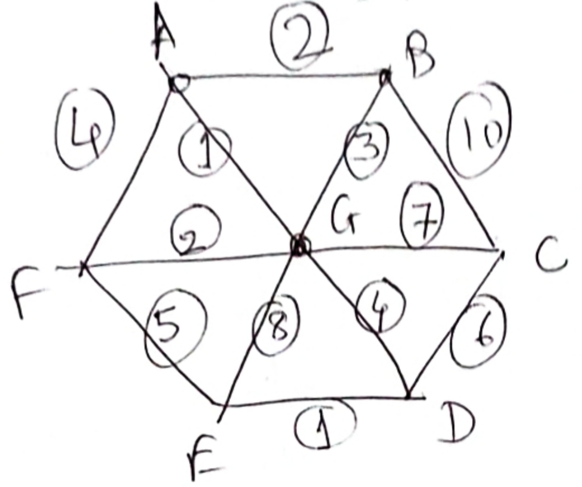
Here $2^{30} \equiv 2 \cdot 2^{29} \equiv (1)$ (mod 31)

∴ $\boxed{2^{-1} \text{ (mod 31)} = 2^{29}}$

$\boxed{2^{-1} \text{ (mod 31)} = 2^{29}}$

Q. 3 (c) Using Kruskal's Algorithm to construct minimum spanning tree of following Graph.

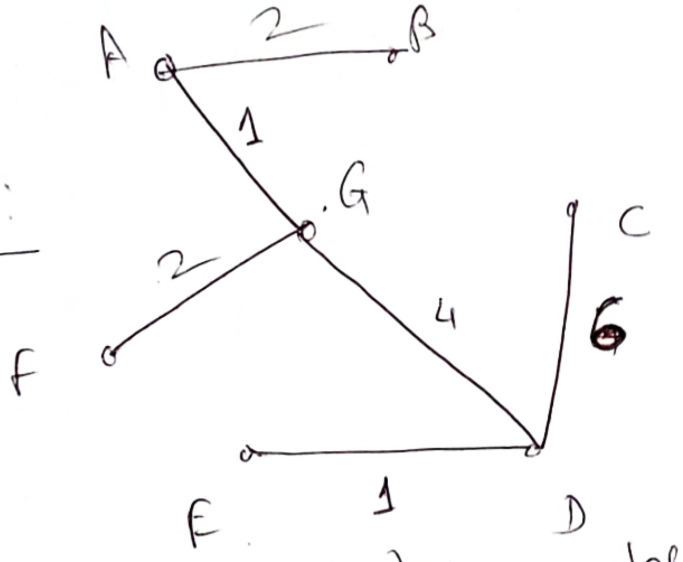
By Kruskal's Algorithm.



weights in Ascending order

minimum wt 1 ✓ Not forming loop
min. wt 1 ✓ Not forming loop
min wt 2 ✓ Not forming loop
min wt 2 ✓ Not forming loop
3 × forming loop
GD 4 ✓ Not forming loop
AF 4 × forming loop
5 × forming loop
6 ✓ Not forming loop
Forming loop 7 ×
forming loop 8 ×
forming loop 10 ×

Graph H:



The above Graph H is a tree which is minimum spanning tree with weight = 1+1+2+2 +4+6 = 16

Q. 3(d) Based on the following data to determine if there is a relation between literacy and smoking

	Smokers	Non-smokers	Total
Literates	O = 83 E=71	O = 57 E=69	140
Illiterates	O = 45 E=57	O = 68 E=56	113
Total	128	125	253

Ho : There is no relation between literacy & smoking

Ha : there is relation between literacy & smoking.

Observed frequency (O)	Expected frequency (E)	O-E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
83	70.83 ≈ 71	12	144	$\frac{144}{71} = 2.028169$
57	69	-12	144	2.028169
45	57	-12	144	2.028169
68	56	12	144	2.028169
				Sum = 8.112676

NOW,

$$\chi^2_{calc} = \sum \frac{(O-E)^2}{E} = 8.112676$$

with 5% LOS, $\chi^2_{table} = 3.814$

as Degree of freedom = (2-1) × (2-1) = 1×1 = 1

as Degree of freedom = 1,

∴ For 5% LOS, Degree of freedom = 1,

$\chi^2_{table} = 3.814$

$\therefore \chi^2_{(critical)} = \chi^2_{table} = 3.814 < 8.112676 = \chi^2_{calc}$

$\therefore \chi^2_{cal} > \chi^2_{table}$

∴ Ho is rejected ∴ Ha is accepted

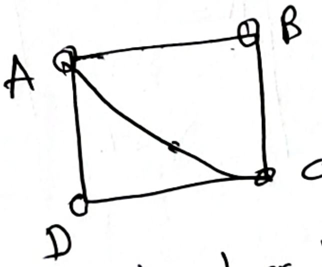
∴ There is relation between literacy & smoking.

Q.4 (a)

(i) Is Every Hamiltonian Graph is Eulerian? Given an example.

Ans:- No,
Every Hamiltonian Graph may not be Eulerian
(is not)

Example



←: Graph G
connected

This Graph has Hamiltonian circuit
A → B → C → D ∴
Graph G is Hamiltonian

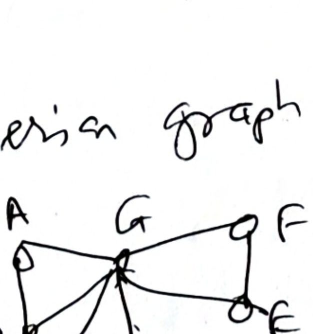
d(A) = 3, d(B) = 2, d(D) = 2 D(C) = degree(C) = 3

since not all vertices are of even degree
the graph G is not Eulerian

(ii) Is Every Eulerian graph Hamiltonian?
Give an example.

Ans: No.
Every Eulerian graph is not Hamiltonian

Ex: Graph H:
is connected



d(A) = 2 = d(B) = d(C)
d(D) = 2 = d(E) = d(F)
d(G) = 6

Since every vertex is of even degree, H is Eulerian

Now, H is not Hamiltonian as every circuit that has all vertices in it contain G more than 2 times

Q. 4(b) To find regression coefficients given that the equations of lines of regression are

$x = 19.13 - 0.87y$ & $y = 11.64 - 0.50x$

$x = 19.13 - 0.87y$

$\Rightarrow b_{xy} = -0.87$

$y = 11.64 - 0.50x$

$\Rightarrow b_{yx} = -0.50$

$\Rightarrow b_{xy} b_{yx} = (-0.87)(-0.50)$

$= 0.475 < 1$

$\therefore b_{xy} = -0.87$

$b_{yx} = -0.50$

if $x = 19.13 - 0.87y$

$\Rightarrow y = \frac{1}{-0.87}(19.13 - x)$

$y = \frac{-19.13}{0.87} + \frac{1}{0.87}x$

$b_{yx} = \cancel{19} \frac{+1}{0.87}$

& $y = 11.64 - 0.50x$

$-0.50x = 11.64 - y$

$x = \frac{11.64}{0.50} + 2y$

$\Rightarrow b_{xy} = 2$

$\therefore b_{xy} \cdot b_{yx} = \frac{1}{0.87} \times 2 > 1$

which is not possible as

$b_{xy} b_{yx} = r^2 \leq 1$

r = correlation coefficient

$\boxed{\text{Q 4 (c)}}$ To Construct an orthonormal basis of $(\mathbb{R}^3 =) R^3$ using Gram Schmidt process

$S = \{(3,0,4), (-1,0,7), (2,9,11)\}$

let $\boxed{u_1 = v_1 =} (3,0,4), = u_1$

$u_2 = (-1,0,7)$ $\quad u = (x_1, y_1, z_1)$

$u_3 = (2,9,11)$ $\quad v = (x_2, y_2, z_2)$

$u \cdot v = x_1x_2 + y_1y_2 + z_1z_2$

∴ To obtain orthogonal basis $\{v_1, v_2, v_3\}$,

∴ step (i). let $\boxed{v_1 = u_1 = (3,0,4) = v_1}$

step (ii) $V_2 = u_2 - \text{Proj}_{u_1} u_2 = u_2 - \text{Proj}_{v_1} u_2$

$= u_2 - \underline{\text{Proj}}_{v_1} u_2$

$= u_2 - \text{Proj}_{v_1} u_2$

$= u_2 - \frac{u_2 \cdot v_1}{v_1 \cdot v_1} v_1$

$= (-1,0,7) - \frac{(-1,0,7)\cdot(3,0,4)}{(3,0,4)\cdot(3,0,4)} (3,0,4)$

$= (-1,0,7) - \frac{25}{25}(3,0,4) = (-1,0,7) - (3,0,4)$

$\boxed{V_2 = (-4,0,3)}$

Here $u_3 \cdot v_1 = (2,9,11)\cdot(3,0,4) = 6+0+44 = 50$

$u_3 \cdot v_2 = (2,9,11)\cdot(-4,0,3) = -8+0+33 = 25$

$v_1 \cdot v_1 = 25 = v_2 \cdot v_2$

step (iii) $V_3 = u_3 - \text{Proj}_{v_1} u_3 - \text{Proj}_{v_2} u_3 = u_3 - \frac{u_3 \cdot v_1}{v_1 \cdot v_1} v_1 - \left(\frac{u_3 \cdot v_2}{v_2 \cdot v_2}\right) v_2$

$= u_3 - \left(\frac{u_3 \cdot v_1}{v_1 \cdot v_1}\right) v_1 - \left(\frac{u_3 \cdot v_2}{v_2 \cdot v_2}\right) v_2$

$= (2,9,11) - \frac{50}{25}(3,0,4) - \frac{(25)}{(25)}(-4,0,3)$

$= (2,9,11) - (6,0,8) - (-4,0,3) = (0,9,0),$

∴ Orthonormal Basis is $\{w_1, w_2, w_3\}$; $w_1 = \frac{v_1}{\|v_1\|} = \frac{v_1}{\sqrt{v_1 \cdot v_1}} = \left(\frac{3}{5}, 0, \frac{4}{5}\right)$, $w_2 = \left(\frac{-4}{5}, 0, \frac{3}{5}\right)$,

$w_3 = v_3/\|w_3\| = (0,1,0)$.

Q.4 (D) A drug was administered to 10 patients and changes in the sugar content in the blood was

10, 8, −6, −4, 2, −8, 6, −5, −3, −6.

Is it reasonable to Believe that the drug has no effect on change of sugar?

Soln: The Null Hypothesis $H_0 : \mu = 0$; No effect

Alternative Hypothesis $H_a : \mu \neq 0$; effect

Changes in sugar content in Blood of 10 patients after administering a drug.

10, 8, −6, −4, 2, −8, 6, −5, −3, −6

Calculation of $\bar{x}$ and s^2

											Sum
$X = x_i$	10	8	−6	−4	2	−8	6	−5	−3	−6	−6
$d_i = x_i + 1$	11	9	−5	−3	3	−7	7	−4	−2	−5	4
$d_i^2 = (x_i - 1)^2$	121	81	25	9	9	49	49	16	4	25	388

$\boxed{a = -1}$

$\boxed{n = 10}$ n = no. of patients = 10

$\bar{x} = a + \frac{\Sigma d_i}{n} = -1 + \frac{4}{10} = -1 + 0.4 = -0.6$

$\Sigma (x_i - \bar{x})^2 = \Sigma d_i^2 - \frac{(\Sigma d_i)^2}{n} = 388 - \frac{4^2}{10}$

$= 288 - 1.6 = 286.4$

$\therefore s^2 = \frac{1}{n} \Sigma (x_i - \bar{x})^2 = \frac{286.4}{10} = 28.64 = \sigma^2$

Calculation of test statistics → As n = 10 < 30, this is small sample, we use t-test.

$t_{cal} = t = \frac{\bar{x} - \mu}{s / \sqrt{n-1}} = \frac{-0.6 - 0}{\sqrt{28.64} / \sqrt{9}} = \frac{-1.8}{\sqrt{28.64}} = -0.3363457$

$\therefore |t_{cal}| = |t| = 0.3363457$. Now $t_{0.05} = t_{table} = 2.262$ for degree of freedom $= n-1 = 10-1 = 9$ $\therefore |t_{cal}| < t_{table}$

$\therefore$ Null Hypothesis is accepted $\therefore \mu = 0$ $\therefore$ It is reasonable to believe that the drug has no effect on change of sugar.